

## Infrared Radiation Parameterizations in Numerical Climate Models

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### ABSTRACT

Parameterizations for infrared radiation (IR) in clear atmospheres can be made fast and accurate by grouping spectral regions with similar radiative properties, and by separating the low pressure region of the atmosphere from the high pressure region. Various approaches are presented in this study to parameterizing the broadband transmission functions for use in numerical climate models. For water vapor and carbon dioxide (CO<sub>2</sub>) bands, the transmission functions are parameterized separately for the middle atmosphere (0.01–30 mb) and for the region below. In the middle atmosphere where the dependence of absorption on pressure and temperature is not strong, the diffuse transmission functions are derived from that at a reference pressure and temperature. In the lower stratosphere and the troposphere, the spectra are grouped into band-center regions and band-wing regions. One-parameter scaling is applied to approximate a nonhomogeneous path with an equivalent homogeneous path, and the diffuse transmittances are either fit by analytical functions or interpolated from precomputed tables.

As opposed to the one-parameter scaling, which applies only to a relatively narrow pressure range, the two-parameter scaling (commonly called the Curtis–Godson approximation) is applied to parameterizing the carbon dioxide (CO<sub>2</sub>) and ozone (O<sub>3</sub>) transmission functions in both the middle and the lower atmosphere. The diffuse transmission functions are simply interpolated from three small precomputed tables. The accuracies of cooling rates in the 15- $\mu$ m band computed using the approximation for both the middle and the lower atmospheres are comparable with that using the parameterizations separately for the middle and the lower atmospheres. The radiative effect of nitrous oxide (N<sub>2</sub>O) and methane (CH<sub>4</sub>) is also examined. Parameterizations are presented for the N<sub>2</sub>O and CH<sub>4</sub> diffuse transmission functions.

### 1. Introduction

The infrared spectrum of the earth–atmosphere system is very complex. A large amount of computer time is required to calculate detailed spectral radiative transfer. In climate (or weather) applications, only the spectrally integrated fluxes are relevant, therefore, band models are used to approximate the time consuming spectral calculations. In the parameterization of infrared radiation using broadband models, analytical functions are developed for spectrally averaged transmittance that fit the results of detailed spectral calculations or laboratory measurements.

Radiative transfer in clear atmospheres depends upon the variations of temperature, pressure, and absorber amount with height. With a few exceptions, which compute transmission functions based upon the deviations of temperature and pressure from reference profiles (e.g., Chou and Kouvaris 1981; Fels and Schwartzkopf 1981), the effects of temperature and

pressure are always taken into account by scaling a nonhomogeneous path to a homogeneous path with an effective temperature and pressure. The relevant quantity to be evaluated for broadband flux calculations is the product of the source (i.e., Planck) and transmission functions integrated over spectral region. It is a function of the effective pressure and temperature of a path, the temperature of source function, and the amount of absorber in a path. A four-dimensional table for this quantity can be precomputed using a detailed line-by-line method, and atmospheric IR fluxes can be accurately computed. In such a case, calculations of atmospheric IR fluxes are straightforward and no band models are required.

A four-dimensional table, however, is cumbersome to use especially when more than one absorber is involved, which enlarges the dimension of the precomputed tables. It is, therefore, necessary to reduce the size of the tables or even to introduce analytical functions to fit the tables (i.e., broadband models) for efficient flux calculations. Thus, parameterization for IR transmission functions is reduced to two major steps: 1) scaling of a nonhomogeneous path and 2) reduction of the size of the precomputed transmission tables (or introducing band models).

The parameterization can be simplified by taking into account the nature of IR radiative transfer in the

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atmosphere, which varies with spectra, heights, and absorbers. For example, water vapor decreases rapidly with height and is most important in affecting cooling in the troposphere, while carbon dioxide ( $\text{CO}_2$ ) is distributed rather uniformly throughout the atmosphere and has a very large effect on the cooling of the stratosphere and the region above. Moreover, the water-vapor spectrum covers practically the entire IR region, while the strongest absorption due to  $\text{CO}_2$  is confined to a relatively narrow band. Within the troposphere the  $\text{CO}_2$  absorption overlaps strongly with the water-vapor absorption. This diversity in radiative transfer makes a universal approach to parameterization very difficult. To be efficient, different forms of parameterization need to be applied to different spectral regions and heights.

The purpose of this study is to demonstrate that scaling of a nonhomogeneous path and computation of transmission functions can be greatly simplified by grouping spectral regions with similar radiative properties and by separating the higher atmosphere from the lower atmosphere. Various forms of parameterizations are given in sections 4, 5, and 6, respectively, for the  $\text{H}_2\text{O}$  bands, the  $15\text{-}\mu\text{m}$   $\text{CO}_2$  band, and the  $9.6\text{-}\mu\text{m}$  ozone ( $\text{O}_3$ ) band. Section 7 describes the parameterization for the transmissions due to nitrous oxide ( $\text{N}_2\text{O}$ ) and methane ( $\text{CH}_4$ ). A summary is given in section 8. This study concerns absorption due to gases only. The effects of clouds, aerosols, and surface properties on IR radiative transfer are not addressed.

## 2. Line-by-line calculations and atmospheric models

Parameterizations for IR transmission functions in this study are based on line-by-line calculations using the 1982 version of the Air Force Geophysics Laboratory (AFGL) absorption line parameters (Rothman et al. 1983). Details of the line-by-line calculations are given in Chou and Kouvaris (1986). The absorption coefficients were precomputed at narrow wavenumber intervals for 24 pressures extending from 0.025 mb to the surface with  $\Delta \log_{10}(p) = 0.2$ , and for 3 temperatures (210, 250, and 290 K), where  $p$  is the pressure. The size of the intervals is  $0.01 \text{ cm}^{-1}$  for the  $\text{H}_2\text{O}$ ,  $\text{N}_2\text{O}$ , and  $\text{CH}_4$  absorption bands,  $0.005 \text{ cm}^{-1}$  for the  $9.6\text{-}\mu\text{m}$   $\text{O}_3$  band, and  $0.002 \text{ cm}^{-1}$  for the  $15\text{-}\mu\text{m}$   $\text{CO}_2$  band. The choice of various spectral resolutions is based on the absorption line width of the regions (both in height and wavenumber) where cooling is important. The cooling due to  $\text{H}_2\text{O}$ ,  $\text{N}_2\text{O}$ , and  $\text{CH}_4$  is important in the troposphere and lower stratosphere where the absorption line is wider than that at higher altitudes, and a larger spectral interval is chosen. On the other hand, the Doppler line width in the  $9.6\text{-}\mu\text{m}$   $\text{O}_3$  band is nearly twice as large as that in the  $15\text{-}\mu\text{m}$   $\text{CO}_2$  band. Therefore, the size of the spectral interval in the  $9.6\text{-}\mu\text{m}$   $\text{O}_3$  band is chosen to be about twice as large as that in the  $15\text{-}\mu\text{m}$   $\text{CO}_2$  band. The absorption coefficient at conditions other than those 24 pressures and 3 temperatures can

be interpolated linearly with pressure and exponentially with temperature. The absorption line was assumed to follow the Voigt function. Below the 10-mb level, the line was cut off at a wavenumber either 385 times the Lorentz half-width or  $6 \text{ cm}^{-1}$  from the center, whichever is smaller. Above the 10-mb level, the line was cut off at a wavenumber 385 times the Lorentz half-width at 10 mb. The diffuse transmittance was computed from beam transmittance using four-point Gaussian quadrature.

Fluxes and cooling rate are computed for a midlatitude summer atmosphere and a subarctic winter atmosphere taken from McClatchey et al. (1972). For the midlatitude summer case, the atmosphere between the 0.01-mb level and the surface is divided into 86 layers. The thickness of a layer is 20 mb below the 100-mb level and  $\Delta \log_{10} p = 0.1$  above. Temperature and  $\text{O}_3$  concentration at the middle of each layer are interpolated linearly with the logarithm of pressure from the data given in McClatchey et al. Below the tropopause (180 mb), the logarithm of specific humidity is interpolated linearly with the logarithm of pressure. Above the tropopause, the specific humidity is set to  $4 \times 10^{-6} \text{ g g}^{-1}$ . For the subarctic winter case, the atmosphere is divided into 60 layers. Temperature, humidity, and ozone concentration for each layer are identical with that used in Ridgway et al. (1990). Carbon dioxide, nitrous oxide, and methane concentrations are set to 300, 0.28, and 1.75 ppmv, respectively, throughout the atmosphere.

Our experience with various line-by-line calculations shows that if the same molecular line data are used, the discrepancy in the flux calculations is primarily caused by the treatment of line shapes in the far wings. In the  $340\text{--}540 \text{ cm}^{-1}$  spectral region, for example, line cutoffs at 5 and at  $10 \text{ cm}^{-1}$  cause a difference of  $2 \text{ W m}^{-2}$  in the downward surface flux for the midlatitude summer atmosphere. Line-by-line calculations of the fluxes at the top of the atmosphere and at the surface for the two atmospheres are given in Table 1. Compared to the calculations of Ridgway et al. (1990), the total fluxes are larger by 2–4  $\text{W m}^{-2}$  at the top of the atmosphere and smaller by 3–5  $\text{W m}^{-2}$  at the surface. The discrepancy is primarily caused by the following factors: 1) the current line-by-line calculations use a variable line cutoff with a nominal value of  $6 \text{ cm}^{-1}$  in the troposphere, while the calculations of Ridgway et al. (1990) use a fixed cutoff of  $10 \text{ cm}^{-1}$ ; 2) the  $\text{H}_2\text{O}$  continuum absorption in the  $400\text{--}500 \text{ cm}^{-1}$  region is included in the calculations of Ridgway et al. (1990), but not in the current calculations; 3) absorption due to  $\text{CO}_2$  in the  $9.4\text{-}\mu\text{m}$  and  $10.4\text{-}\mu\text{m}$  bands are included in the study of Ridgway et al. (1990), but not in the current study.

## 3. Simplifications for transmission calculations

In the absence of scattering, the upward flux  $F\uparrow(p)$  and the downward flux  $F\downarrow(p)$  at the pressure level  $p$

TABLE 1. Upward fluxes at the top of the atmosphere  $F\uparrow(\text{top})$  and the downward fluxes at the surface  $F\downarrow(\text{sfc})$ . The  $\text{CO}_2$  concentration is set to 300 ppmv. The units of fluxes are watts per square meter.

	$F\uparrow(\text{top})$	$F\downarrow(\text{sfc})$
H <sub>2</sub> O region (0–540, 800–980, 1100–3000 cm <sup>-1</sup> )		
Midlatitude summer		
Line-by-line	204.4	220.4
Parameterization	207.9	219.1
Subarctic winter		
Line-by-line	142.8	103.1
Parameterization	143.4	101.5
15- $\mu\text{m}$ region (540–800 cm <sup>-1</sup> ), H <sub>2</sub> O + CO <sub>2</sub>		
Midlatitude summer		
Line-by-line	68.0	106.6
Parameterization <sup>a</sup>	67.8	108.8
Parameterization <sup>b</sup>	68.3	108.7
Subarctic winter		
Line-by-line	51.5	53.1
Parameterization <sup>a</sup>	51.8	52.1
Parameterization <sup>b</sup>	51.8	51.5
9.6- $\mu\text{m}$ region (980–1100 cm <sup>-1</sup> ), H <sub>2</sub> O + O <sub>3</sub>		
Midlatitude summer		
Line-by-line	22.3	12.8
Parameterization	21.7	12.9
Subarctic winter		
Line-by-line	10.6	3.2
Parameterization	10.8	3.3
Total (0–3000 cm <sup>-1</sup> ), H <sub>2</sub> O + CO <sub>2</sub> + O <sub>3</sub>		
Midlatitude summer		
Line-by-line	294.7	339.8
Parameterization <sup>a</sup>	298.2	339.8
Subarctic winter		
Line-by-line	204.8	159.4
Parameterization <sup>a</sup>	204.8	156.8

<sup>a</sup> Using table lookup and Eq. (33) for computing the  $\text{CO}_2$  transmission function.

<sup>b</sup> Using the one-parameter scaling and Eqs. (34) and (35) for computing the  $\text{CO}_2$  transmission function.

can be computed from

$$F\uparrow(p) = \int d\nu \left\{ B_\nu(T_s) \tau_\nu(p, p_s) + \int_{p_s}^p B_\nu[T(p')] [\partial \tau_\nu(p, p') / \partial p'] dp' \right\}, \quad (1)$$

and

$$F\downarrow(p) = \int d\nu \left\{ \int_0^p B_\nu[T(p')] [\partial \tau_\nu(p, p') / \partial p'] dp' \right\}, \quad (2)$$

where  $\tau_\nu(p, p')$  is the diffuse transmittance between the pressure levels  $p$  and  $p'$ ,  $B_\nu(T)$  is the Planck flux,  $\nu$  is the wavenumber,  $T$  is the temperature, and the

subscript  $s$  denotes the surface. The diffuse transmittance is given by

$$\tau_\nu(p, p') = 2 \int_0^1 \exp \left\{ - \int_{p'}^p k_\nu[p'', T(p'')] \times q(p'') dp'' / (g\mu) \right\} \mu d\mu, \quad (3)$$

where  $\mu$  is the cosine of the angle departing from the vertical,  $k_\nu$  is the absorption coefficient,  $q$  is the absorber mixing ratio, and  $g$  is the gravitational acceleration.

According to (1) and (2), the relevant quantity in the flux and cooling rate calculations is

$$\int \tau_\nu(p, p') B_\nu(T') d\nu = \tau(p, p', T') \int B_\nu(T') d\nu, \quad (4)$$

where  $\tau(p, p', T')$  is the Planck-weighted diffuse transmission function given by

$$\tau(p, p', T') = \int \tau_\nu(p, p') B_\nu(T') d\nu / \int B_\nu(T') d\nu, \quad (5)$$

and  $T'$  is the temperature at the pressure level  $p'$ .

For a homogeneous layer with constant pressure and temperature, the Planck-weighted diffuse transmission function can be precomputed using line-by-line methods. Thus, the parameterization for the transmission functions becomes a problem of scaling a nonhomogeneous path to an equivalent homogeneous path. The accuracy of scaling approximation can be enhanced if specific simplifications are applied based upon the range of atmospheric conditions, the spectral property of absorption bands, and the absorber involved.

#### a. Two-parameter scaling

This approach, commonly called Curtis–Godson approximation, is to scale a nonhomogeneous path to a homogeneous path with an effective temperature and pressure, so that the transmission of a path can be computed from the effective temperature and pressure. The effective temperature and pressure are usually computed by weighting the absorber amount  $w$  along a path,

$$p_{\text{eff}} = \int p dw / \int dw, \quad (6)$$

$$T_{\text{eff}} = \int T dw / \int dw. \quad (7)$$

It has been well recognized (e.g., Wu 1980; Chou and Kouvaris 1990) that the contribution to the cooling rate is primarily from adjacent layers. The pressure and temperature variations among adjacent layers are usually small, and the simple scaling approximations of (6) and (7) can be applied for accurate cooling rate calculations. For distant layers the scaling approxi-

mations may not be appropriate, but the effect on flux and cooling rate calculations is small.

With the use of the two-parameter scaling, the Planck-weighted diffuse transmittance becomes a function of the effective temperature and pressure, the absorber amount, and the temperature of the layer from where radiation originates, i.e.,  $\tau(p, p', T') \approx \tau(p_{eff}, T_{eff}, w, T')$ . This function can be accurately precomputed using line-by-line methods. The ranges of the variables  $p_{eff}$ ,  $T_{eff}$ ,  $w$ , and  $T'$  are large, especially for  $p_{eff}$  and  $w$ , which span several orders of magnitude. It is, therefore, not practical to use large precomputed tables. It is also difficult to fit the large tables with analytical functions. However, the parameterization can be simplified if it is applied to absorption bands where the spectral variation of the Planck function is moderate. In such a case, the Planck-weighted diffuse transmission function can be approximated by

$$\tau(p, p', T') \approx \int \tau_\nu(p, p') B_\nu(T_0) d\nu \bigg/ \int B_\nu(T_0) d\nu \approx \tau(p_{eff}, T_{eff}, w; T_0), \quad (8)$$

where  $T_0$  is a reference value chosen to be 250 K in this study. The transmittance is thus reduced to a function of three variables. Because  $\tau(p_{eff}, T_{eff}, w; T_0)$  varies smoothly with  $T_{eff}$ , it can be further simplified by separating  $T_{eff}$  from the function.

#### b. One-parameter scaling

When fluxes are to be computed for a very wide spectral range where the Planck function varies significantly and where more than one absorber is involved, the number of independent variables in the Planck-weighted transmission function increases. It becomes extremely difficult to evaluate this function. In such a case, the number of independent variables can be reduced by using a one-parameter scaling. Instead of scaling the temperature and pressure along a path as in the two-parameter scaling, the absorption coefficient is extrapolated from the absorption coefficient at  $p_r$  and  $T_r$ ,

$$k_\nu(p, T) = k_\nu(p_r, T_r) f(p, T; p_r, T_r), \quad (9)$$

where  $f$  is the scaling function. From (3), the diffuse transmission function becomes

$$\tau_\nu(w_{eff}) = 2 \int_0^1 \exp[-w_{eff} k_\nu(p_r, T_r) / \mu] \mu d\mu, \quad (10)$$

where  $w_{eff}$  is the scaled absorber amount given by

$$w_{eff} = \int_{p'}^p q(p) f[p, T(p); p_r, T_r] dp / g. \quad (11)$$

The Planck-weighted diffuse transmission function is now dependent only upon  $w_{eff}$  and  $T'$ ,

$$\begin{aligned} \tau(p, p', T') &\approx \int \tau_\nu(w_{eff}) B_\nu(T') d\nu \bigg/ \int B_\nu(T') d\nu \\ &= \tau(w_{eff}, T'). \end{aligned} \quad (12)$$

A number of forms for the scaling function have been chosen by various investigators (e.g., Chou and Arking 1980; Manabe and Strickler 1964; Morcrette et al. 1986; Shi 1984). A simple scaling function adopted by Chou and Arking (1980) is given by

$$f(p, T; p_r, T_r) = (p/p_r)^m \exp[r_1(T - T_r) + r_2(T - T_r)^2] \quad (13)$$

where  $m$  is an empirical constant. In determining the constants  $r_1$  and  $r_2$ , the transmission function  $\tau(w, p_r, T)$  is first computed at a number of temperatures and the absorber amount  $w$ . Regression is then applied to determine  $r_1$  and  $r_2$  such that the difference between  $\tau(w, p_r, T)$  and  $\tau(w_{eff}, p_r, T_r)$  is minimized.

Compared to the two-parameter scaling, the number of independent variables in the Planck-weighted transmission function is reduced by two. Because the absorption coefficient is extrapolated according to (9), the error in cooling rate calculations using the one-parameter scaling might be large at heights where pressure and temperature are far from  $p_r$  and  $T_r$ . Generally, this scaling approximation can only be applied to spectral regions with similar optical properties. The cooling contributed by these spectral regions usually is confined to a narrow region in the atmosphere wherein the temperature and pressure variations are small. Errors of the one-parameter scaling can be minimized if the pressure and temperature typical of the height of peak cooling are chosen for  $p_r$  and  $T_r$ . It is noticed that for those parameterizations using the one-parameter scaling, the reference pressure and temperature are commonly chosen to be the standard conditions (1013 mb and 273 K). It is clear from (9) that this choice for the reference pressure and temperature is inappropriate except for the first few kilometers above the surface, since far from the surface large cooling rate errors could occur.

#### c. Grouping of spectral regions

Spectral regions with similar optical properties can be grouped together to simplify flux calculations. The intensity of a molecular line is only weakly dependent on temperature near the center of an absorption band, but is strongly dependent on temperature in the wings. Furthermore, the peak of a cooling profile is located at higher altitudes near the band centers and at lower altitudes in the band wings. Therefore, the effects of temperature and pressure variations along a path can be more accurately taken into account if parameterizations are applied separately to the band centers and the band wings.

#### d. Separation of regions in the atmosphere

Broadening of a molecular line is dominated by the pressure effect in the troposphere and by the Doppler effect in the middle atmosphere ( $\approx 0.01\text{--}3$  mb). Some parameterizations that include both effects are based on pressure broadening. The Doppler effect is included by empirically adjusting the pressure scaling. In the middle atmosphere, the Doppler broadening is generally more important than the pressure broadening. Parameterizations based on the pressure broadening are not expected to yield accurate cooling rate in the middle atmosphere. This difficulty can be reduced by separating the middle atmosphere from the lower atmosphere. In doing so, not only the pressure range is narrowed but also the pressure effect is weakened. Thus, the parameterization can be made more accurate.

#### 4. Water-vapor bands

The water-vapor spectrum covers practically the entire infrared region where the Planck function varies significantly with wavenumber and temperature. The effect of the Planck function must be included in computing the broadband transmittance. The Planck-weighted transmittance is a function of four variables;  $p_{eff}$ ,  $T_{eff}$ ,  $w$ , and  $T'$  if the two-parameter scaling is used and only the line absorption is considered. When the continuum absorption is also considered, an extra independent variable is introduced. To evaluate the complicated transmission function, it is a common practice to develop broadband models (e.g., Howard et al. 1956; Ramanathan and Downey 1986; Morcrette et al. 1986) that fit the laboratory or line-by-line calculated transmittances. Because of its multidimensional nature, it can be expected that modeling of the transmission function is very difficult. However, the number of independent variables in the transmission function can be reduced and the parameterization simplified if the middle atmosphere is separated from the region below.

#### a. Troposphere and lower stratosphere

The one-parameter scaling can be applied to heights where pressure and temperature are not much different from  $p_r$  and  $T_r$ . For a spectral range with uniform optical properties, the cooling profile has a narrow spread in height. It is, therefore, advantageous to divide the spectrum into noncontiguous groups with similar optical properties, and apply the one-parameter scaling. The Planck-weighted diffuse transmission function is then approximated by

$$\tau(p, p', T') \approx \tau(w_{eff}, u_{eff}, T'), \quad (14)$$

where  $w_{eff}$  is the scaled water-vapor amount for line absorption given by (11), and  $u_{eff}$  is the scaled water-vapor amount for continuum absorption given by

$$u_{eff} = \int_p^{p'} e(p'') \exp\{1800[1/T(p'') - 1/296]\} \times q(p'') dp'' / g, \quad (15)$$

$e$  is the water-vapor partial pressure in atmospheres, and  $T$  is in kelvins.

The values of  $p_r$  and  $T_r$  are chosen to be the typical pressure and temperature near the height of peak cooling. Chou and Arking (1980) and Chou (1984) grouped the water-vapor spectrum into two noncontiguous regions, corresponding to band centers ( $0\text{--}340$  and  $1380\text{--}1900$   $\text{cm}^{-1}$ ) and band wings ( $340\text{--}540$ ,  $800\text{--}980$ ,  $1100\text{--}1380$ , and  $1900\text{--}3000$   $\text{cm}^{-1}$ ). The absorption in the band-center region is strong, and the peak of the cooling profile is located at upper troposphere. The reference temperature and pressure are, therefore, chosen to be representative of that region. On the other hand, the cooling profile for the band-wing region peaks in the lower troposphere, and the values of  $p_r$  and  $T_r$  are chosen to be larger. The spectral range, the reference pressure and temperature, together with the values of  $m$ ,  $r_1$ , and  $r_2$  for scaling the water-vapor amount using (11) are given in Table 2.

A three-dimensional table for  $\tau(w_{eff}, u_{eff}, T')$  has been precomputed using the line-by-line method and the continuum absorption coefficient derived from the curve given by Roberts et al. (1976). Because  $\tau(w_{eff},$

TABLE 2. Parameters for the one-parameter scaling, Eq. (13) in various absorption bands.

Spectral range ( $\text{cm}^{-1}$ )	$p_r$ (mb)	$T_r$ (K)	$m$	$r_1$ ( $\text{K}^{-1}$ )	$r_2$ ( $\text{K}^{-2}$ )
Water vapor					
0–340	275	225	1.0	0.005	0.0
1380–1900					
340–540	550	256	1.0	0.016	0.0
800–980					
1100–1380					
1900–3000					
540–800	550	256	1.0	0.016	0.0
560–615	500	250	1.0	0.016	$-5.5 \times 10^{-5}$
980–1100	550	256	1.0	0.016	0.0
1135–1215	500	250	0.5	0.019	$-6.0 \times 10^{-5}$
1215–1380	500	250	1.0	0.009	0.0
CO <sub>2</sub>					
560–615	300	250	0.50	0.016	$-7.0 \times 10^{-5}$
620–720	30	240	0.85	0.009	$3.9 \times 10^{-5}$
540–620	300	240	0.50	0.025	$-1.4 \times 10^{-5}$
720–800					
N <sub>2</sub> O					
560–615	300	250	0.5	0.0	0.0
1135–1215	500	250	0.5	0.005	0.0
1215–1340	500	250	0.5	0.0	$2.1 \times 10^{-4}$
CH <sub>4</sub>					
1215–1380	500	250	0.5	0.0	0.0

Note: The parameters for the CO<sub>2</sub> band are slightly different from that given in Chou and Peng (1983) due to the use of different versions of the AFGL line parameters.

$u_{eff}, T')$  is a smooth function of  $T'$ , it is further reduced to

$$\tau(w_{eff}, u_{eff}, T') = \tau(w_{eff}, u_{eff}; T_0)[1 + \alpha(w_{eff}, u_{eff}) \times (T' - T_0) + \beta(w_{eff}, u_{eff})(T' - T_0)^2], \quad (16)$$

where  $\alpha(w_{eff}, u_{eff})$  and  $\beta(w_{eff}, u_{eff})$  are regression coefficients. Using  $T_0 = 250$  K, the error in  $\tau(w_{eff}, u_{eff}, T')$  due to the curve fitting is  $<0.002$ . In the band-center region ( $0-340$  and  $1380-1900$   $\text{cm}^{-1}$ ), molecular line absorption is strong, and continuum absorption can be neglected. The coefficients  $\alpha$  and  $\beta$  are only functions of  $w_{eff}$ . In computing the diffuse transmittance  $\tau(w_{eff}, u_{eff}, T')$  from (16), values of  $\tau(w_{eff}, u_{eff}; T_0)$ ,  $\alpha(w_{eff}, u_{eff})$ , and  $\beta(w_{eff}, u_{eff})$  can be either interpolated from precomputed tables (confer Chou 1984) or computed from functions fitting to the precomputed tables (confer Harshvardhan et al. 1987). Because the values of  $w_{eff}$  and  $u_{eff}$  change in the same direction, only a narrow range in the  $w_{eff}-u_{eff}$  domain is relevant in atmospheric flux calculations. It is found that, within the relevant range of  $w_{eff}$  and  $u_{eff}$ , values of  $\tau(w_{eff}, u_{eff}; T_0)$ ,  $\alpha(w_{eff}, u_{eff})$ , and  $\beta(w_{eff}, u_{eff})$  can be accurately fit by the following equations. In the band-center region,

$$\tau(w_{eff}; T_0) = \exp[-7790w_{eff}/(1 + 1340w_{eff}^{0.59} + 550w_{eff})], \quad (17)$$

$$\alpha(w_{eff}) = 10^{-4} \times \exp[5.18 + 0.51 \ln w_{eff}], \quad (18)$$

$$\beta(w_{eff}) = 10^{-6} \times \exp[4.61 + 0.71 \ln w_{eff} + 0.014(\ln w_{eff})^2]. \quad (19)$$

In the band-wing region,

$$\tau(w_{eff}, u_{eff}; T_0) = \exp[-230w_{eff}/(1 + 200w_{eff}^{0.6} + 130w_{eff}) - a_0(w_{eff})u_{eff}], \quad (20)$$

$$\alpha(w_{eff}, u) = \alpha(w_{eff}, 0) + a_1(w_{eff})u_{eff}, \quad (21)$$

$$\beta(w_{eff}, u_{eff}) = \beta(w_{eff}, 0) + 1.4 \times 10^{-4}u_{eff}, \quad (22)$$

where

$$a_0(w_{eff}) = 17.51 - 2.51 \ln w_{eff} - 0.046(\ln w_{eff})^2 \quad (23)$$

$$a_1(w_{eff}) = 10^{-4} \times [587 - 102 \ln w_{eff} - 35(\ln w_{eff})^2] \quad (24)$$

$$\alpha(w_{eff}, 0) = 10^{-4} \times \exp[3.77 + 0.174 \ln w_{eff} + 0.032(\ln w_{eff})^2], \quad (25)$$

$$\beta(w_{eff}, 0) = -10^{-6} \times \exp[2.65 + 0.25 \ln w_{eff} + 0.0245(\ln w_{eff})^2]. \quad (26)$$

The units of  $w_{eff}$  and  $u_{eff}$  are grams per square centimeter ( $\text{g cm}^{-2}$ ).

### b. Middle atmosphere

Computations of cooling rate in the middle atmosphere can be made easier and more accurate by sep-

arating the middle atmosphere from the lower atmosphere. There are several advantages in this approach: 1) continuum absorption is negligible because of a small water-vapor concentration; 2) not only is the range in pressure reduced, but also the pressure dependency of absorption is weakened; and 3) The contribution to middle-atmospheric cooling is predominantly from the regions near band centers where absorption is a weak function of temperature. Factors 2) and 3) reduce the difficulty in accounting for the effects of pressure and temperature on absorption. It is clear that the pressure scaling in (13) cannot be used in the middle atmosphere where the line broadening is dominated by the Doppler effect and where the pressure is far from  $p_r$ . Therefore, the two-parameter scaling is used in the middle atmosphere.

The  $\text{H}_2\text{O}$  cooling rate in the middle atmosphere is primarily due to the spectral regions  $0-540$  and  $1380-1900$   $\text{cm}^{-1}$ . Therefore, parameterization is only for these spectral regions. For these two spectral regions, the contribution to the cooling from the latter is much smaller than that from the former because of the smaller Planck function. In the water vapor rotational band ( $0-540$   $\text{cm}^{-1}$ ), the shape of  $B$ , varies rather uniformly with temperature. As a result, the Planck-weighted transmission function  $\tau(p_{eff}, T_{eff}, w, T')$  depends only weakly on  $T'$  and can be approximated by  $\tau(p_{eff}, T_{eff}, w; T_0)$ .

Because the transmittance is not a strong function of pressure and temperature in the middle atmosphere, it is reasonable to expect that  $\tau(p_{eff}, T_{eff}, w; T_0)$  can be extrapolated from the transmittance at a reference pressure and temperature  $\tau(p_r, T_r, w; T_0)$ . The function  $\tau(p_r, T_r, w; T_0)$  depends only on  $w$  and can be either interpolated from precomputed values or fit by a simple function. It is found that, in the  $0-540$  and  $1380-1900$   $\text{cm}^{-1}$  regions, the logarithm of the diffuse absorptance computed at  $p_r = 0.25$  mb and  $T_r = T_0 = 250$  K can be fit well by

$$\ln[1 - \tau(p_r, T_r, w; T_0)] = -2.5 + 0.26 \ln w - 0.01(\ln w)^2. \quad (27)$$

The units of  $w$  are grams per square centimeter ( $\text{g cm}^{-2}$ ). The relative error in the absorptance computed from (27) is  $<5\%$  for  $w$  in the range from  $10^{-7}$  to  $0.1$   $\text{g cm}^{-2}$ .

In the middle atmosphere, an absorption line has a Doppler core and Lorentzian wings. Absorption is dependent on  $w$  in the core and on  $(wp)$  in the wings. The diffuse transmittance is thus extrapolated with pressure according to

$$\tau(p_{eff}, T_r, w; T_0) = \tau(p_r, T_r, w; T_0)[f(w) + g(w, p_{eff})], \quad (28)$$

where  $f$  and  $g$  are functions that fit the intercepts and the slopes of the function  $[\tau(p_{eff}, T_r, w; T_0)/\tau(p_r, T_r, w; T_0)]$ ,

$$f(w) = 0.9999147 + 0.07489w^{0.4} + 1.8286w^{0.8}, \quad (29)$$

and

$$g(w, p_{eff}) = (0.0000826 - 0.07124w^{0.4} - 1.807w^{0.8})(p_{eff}/p_r)^{0.4}. \quad (30)$$

Finally, transmittance is a weak function of temperature in the center region of the absorption bands and can be extrapolated using the following equation;

$$\tau(p_{eff}, T_{eff}, w; T_0) = 1 - [1 - \tau(p_{eff}, T_r, w; T_0)] \times \exp[0.00406(T_{eff} - T_r)]. \quad (31)$$

In summary, the Planck-weighted diffuse transmission function  $\tau(p, p', T')$  in the 0–540 and 1380–1900  $\text{cm}^{-1}$  region is approximated by  $\tau(p_{eff}, T_{eff}, w; T_0)$ , which in turn is computed from (27)–(31).

### c. Fluxes and cooling rate

The validity of the parameterizations is shown in Fig. 1 for the troposphere and in Fig. 2 for the middle atmosphere of the midlatitude summer case. Solid curves are line-by-line calculations. The dashed curves in Figs. 1 and 2 are computed, respectively, using the tropospheric parameterization and the middle-atmospheric parameterization. The cooling rate errors of the parameterizations are shown in Fig. 3. It can be seen in Fig. 3 that the parameterization for the middle atmosphere introduces an error of  $<0.1^\circ\text{C day}^{-1}$ , while the parameterization for the lower atmosphere introduces an error of  $<0.15^\circ\text{C day}^{-1}$ . Fluxes at the top of the atmosphere and the surface computed using the line-by-line method and the tropospheric parameterization are shown in Table 1 for both the midlatitude summer atmosphere and the subarctic winter atmosphere. Compared to line-by-line calculations the error is  $<2\%$ .

For a numerical climate model including both the middle atmosphere and the troposphere, the cooling profile can be obtained by merging the results of the two parameterizations. As can be seen in Fig. 3, the heights where both parameterizations can be applied overlap significantly (10–80 mb). The boundary for merging the cooling profiles can be chosen near the 30-mb level.

## 5. The 15- $\mu\text{m}$ $\text{CO}_2$ band

The radiative transfer in the 15- $\mu\text{m}$   $\text{CO}_2$  band is important throughout the entire atmosphere. It is difficult to devise a parameterization for  $\text{CO}_2$  absorption applicable to the atmosphere where two different line broadening mechanisms operate in two different regions. Parameterizations using broadband models usually work well in the lower portion of the atmosphere (e.g., Kiehl and Ramanathan 1983) where the Doppler broadening of an absorption line is not im-

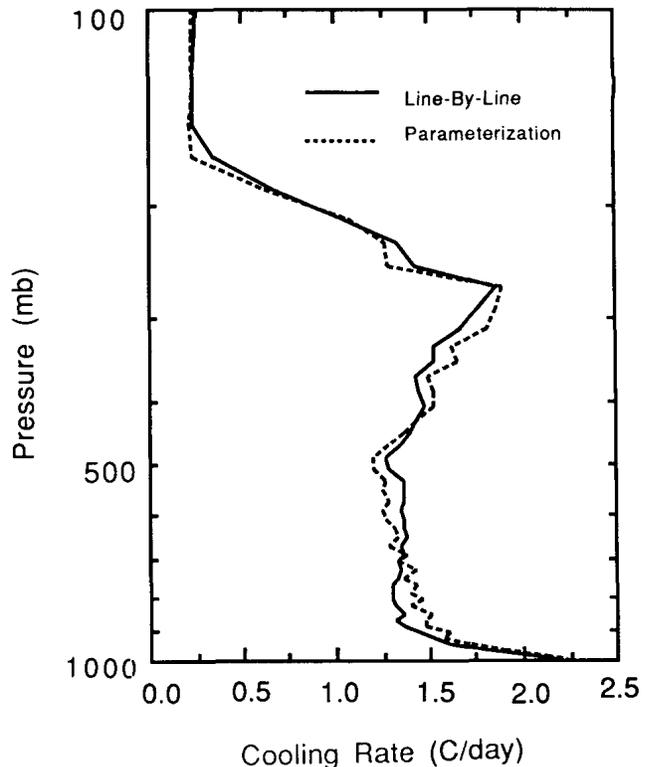


FIG. 1. Tropospheric cooling rate profiles in the water-vapor bands (0–540, 800–980, 1100–3000  $\text{cm}^{-1}$ ) for a midlatitude summer atmosphere. The water vapor specific humidity above the tropopause (180 mb) is set to  $4 \times 10^{-6} \text{ g g}^{-1}$ .

portant. In the middle atmosphere where the Doppler effect is dominant and where computations of the fractional absorptance accurate to the fifth digit are often required (due to the small mass involved), the accuracy of those broadband models in the middle atmosphere remains to be determined. Ou and Liou (1983) fit the line-by-line calculated transmission data of Fels and Schwartzkopf (1981) by two polynomial functions separately for two ranges of the  $\text{CO}_2$  path length in the middle atmosphere. This parameterization is applicable only to the middle atmosphere. Because the  $\text{CO}_2$  concentration can be assumed constant throughout the atmosphere, the transmission function can be accurately computed based upon the deviation of temperature from a reference atmosphere (e.g., Chou and Kouvaris 1981; Fels and Schwartzkopf 1981). However, this approach becomes very complicated if the transmission function is to be computed for a  $\text{CO}_2$  concentration different from the reference value.

### a. A general approach

The 15- $\mu\text{m}$   $\text{CO}_2$  band covers a relatively small spectral range ( $\approx 540\text{--}800 \text{ cm}^{-1}$ ). Line-by-line calculations show that the transmittance is not a strong function of the Planck flux, and the Planck-weighted diffuse trans-

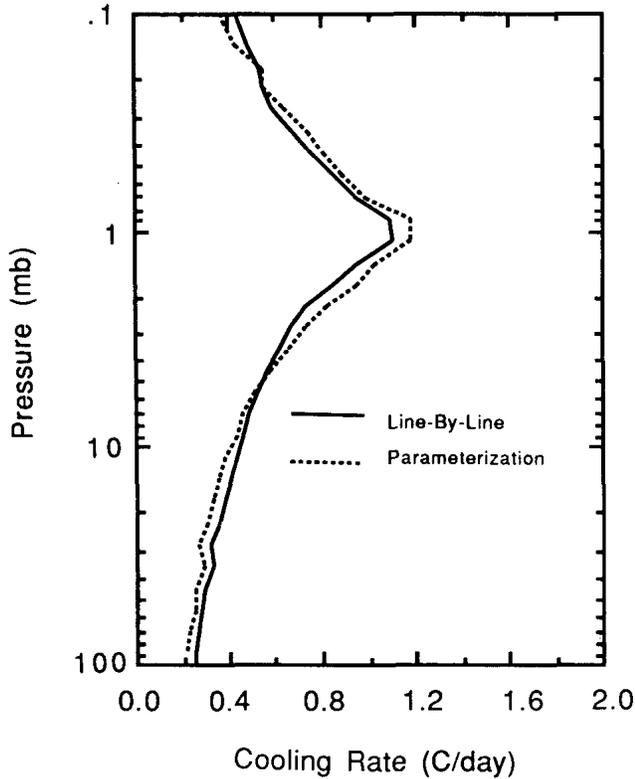


FIG. 2. Middle atmospheric cooling rate profiles in the water-vapor bands (0–540, 1380–1900  $\text{cm}^{-1}$ ) for a midlatitude summer atmosphere. The water-vapor specific humidity above the tropopause (180 mb) is set to  $4 \times 10^{-6} \text{ g g}^{-1}$ .

mittance can be approximated by

$$\tau(p, p', T') \approx \int \tau_\nu(p, p') d\nu = \tau(p, p'). \quad (32)$$

When the two-parameter scaling is used, the transmittance becomes a function only of  $p_{eff}$ ,  $T_{eff}$ , and  $w$ . Because the transmittance is a smooth function of temperature, Chou and Kouvaris (1990) have found that it can be further simplified by using the following equation,

$$\tau(p_{eff}, T_{eff}, w) = \tau(p_{eff}, T_r, w) [1 + \alpha(p_{eff}, w) \times (T_{eff} - T_r) + \beta(p_{eff}, w)(T_{eff} - T_r)^2], \quad (33)$$

where  $\alpha(p_{eff}, w)$  and  $\beta(p_{eff}, w)$  are derived by fitting the line-by-line calculated transmittances and are tabulated in Chou and Kouvaris (1990). The transmission functions can be easily derived from the precomputed tables of  $\tau(p_{eff}, T_r, w)$ ,  $\alpha(p_{eff}, w)$ , and  $\beta(p_{eff}, w)$  and Eq. (33). Because the two-parameter scaling is used, the parameterization can be applied to both the troposphere and the middle atmosphere.

*b. Troposphere and lower stratosphere*

In many climate applications, such as surface energy budget calculations and certain numerical climate

models, calculations of radiative transfer in the middle atmosphere is not required. The range of pressure is narrowed, and only the pressure effect is important in broadening an absorption line. The parameterization can be simplified by using the one-parameter scaling.

In studying the  $\text{CO}_2$  greenhouse effect using a multilayer energy balance climate model, Chou and Peng (1983) have developed a parameterization for the  $\text{CO}_2$  transmission function applicable to the troposphere and the lower stratosphere. As in the water-vapor bands, the 15- $\mu\text{m}$  band is grouped into a band-center region (620–720  $\text{cm}^{-1}$ ) and a noncontiguous band-wing region (540–620 and 720–800  $\text{cm}^{-1}$ ). The one-parameter scaling is used, which reduces the mean transmittance to a function only of the scaled  $\text{CO}_2$  amount  $\tau(p, p') \approx \tau(w_{eff})$ . The parameters in (13) for scaling the  $\text{CO}_2$  amount  $w_{eff}$  are given in Table 2. Chou and Peng (1983) have found that the following function can fit well the line-by-line calculated diffuse transmittances in both the band-center and the band-wing regions,

$$\tau(w_{eff}) = \exp[-aw_{eff}/(1 + bw_{eff}^n)], \quad (34)$$

where  $a$ ,  $b$ , and  $n$  are regression coefficients. Values of  $a$ ,  $b$ , and  $n$  are given in Table 3. The units of  $w_{eff}$  are centimeter-atmospheres  $(\text{cm-atm})_{\text{STP}}$ . The relative er-

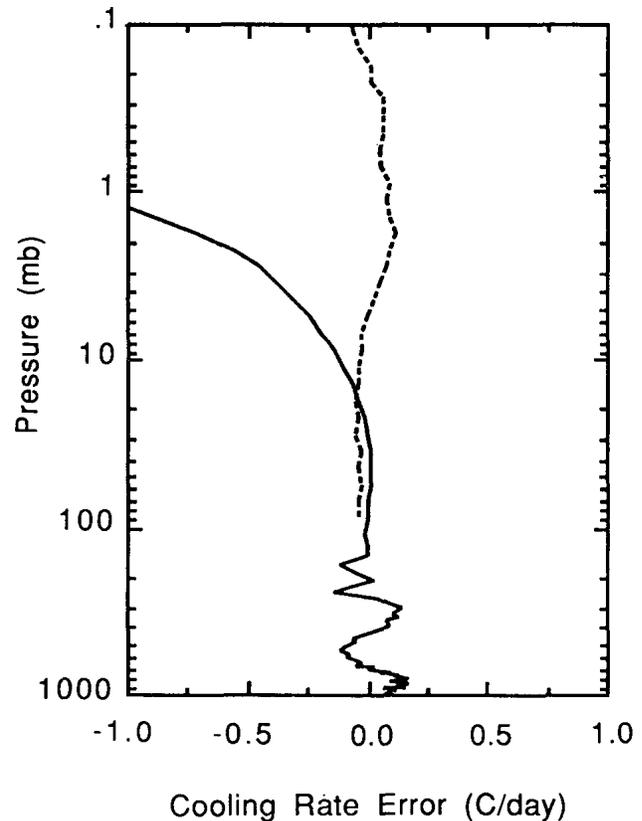


FIG. 3. Errors in the water-vapor cooling rate calculations. The solid curve is for the tropospheric parameterization, and the dashed curve is for the middle-atmospheric parameterization.

TABLE 3. Parameters for computing transmission functions using Eqs. (34) and (42).

Spectral range (cm <sup>-1</sup> )	<i>a</i>	<i>b</i>	<i>n</i>	<i>c</i>	<i>k</i>
Water vapor					
540–800	6.7	16.0	0.60	27.0	0.83
560–615	20.7	31.9	0.55	63.6	0.90
980–1100	0.05	1.47	0.50	8.10	0.94
1135–1215	0.70	4.65	0.55	6.75	0.94
1215–1380	32.5	45.5	0.58		
CO <sub>2</sub>					
560–615	0.023	0.46	0.54		
620–720	3.35	17.0	0.57		
540–620	0.04	0.99	0.58		
720–800					
N <sub>2</sub> O					
560–615	1.19	2.74	0.55		
1135–1215	0.31	0.48	0.57		
1215–1340	4.83	5.45	0.57		
CH <sub>4</sub>					
1215–1380	2.01	5.17	0.58		

ror in the absorptance using (34) is <2% for  $w$  in the range  $10^{-3}$ – $10^3$  (cm-atm)<sub>STP</sub>. The mean CO<sub>2</sub> diffuse transmittance  $\tau$  in the 540–800 cm<sup>-1</sup> region is then computed by weighting the transmittances with the width of respective spectral regions,

$$\tau = 0.385\tau_c + 0.615\tau_w, \quad (35)$$

where the subscripts  $c$  and  $w$  denote the band-center region and the band-wing region, respectively. The parameterization is very simple and efficient but is restricted to the lower stratosphere and the troposphere.

### c. Middle atmosphere

The CO<sub>2</sub> cooling in this region can also be parameterized as in the water-vapor bands. The effective pressure and temperature of a path are computed using the two-parameter scaling, the diffuse transmittance is extrapolated from the reference transmittance,  $\tau(p_r, T_r, w; T_0)$ . The reference transmission function at  $p_r = 0.25$  mb and  $T_r = T_0 = 250$  K is computed using line-by-line methods for the region 540–800 cm<sup>-1</sup> and fit by

$$\tau(p_r, T_r, w; T_0) = \exp[-2w/(1 + 130w^{0.57})]. \quad (36)$$

The units of  $w$  are centimeter-atmospheres (cm-atm)<sub>STP</sub>. Compared to line-by-line calculations, the relative error in the absorptance using (36) is <6% for  $w < 50$  (cm-atm)<sub>STP</sub>.

The diffuse transmittance at pressure  $p_{eff}$  is extrapolated from the reference transmittance according to (28). The functions  $f$  and  $g$  in (28) are determined by fitting to the intercepts and the slopes of the function  $[\tau(p_{eff}, T_r, w; T_0)/\tau(p_r, T_r, w; T_0)]$ ,

$$f(w) = 0.999914 + 0.00613w^{0.55}, \quad (37)$$

and

$$g(w, p_{eff}) = \{[-5.7985 + 0.1767 \ln w + 0.0851(\ln w)^2] \times 10^{-3}\} (p_{eff}/p_r)^{0.55}. \quad (38)$$

The diffuse transmittance at the temperature  $T_{eff}$  is then extrapolated from

$$\tau(p_{eff}, T_{eff}, w; T_0) = 1 - [1 - \tau(p_{eff}, T_r, w; T_0)] \times \exp[0.0054(T - T_r)], \quad (39)$$

where the unit of temperature is Kelvins. Finally,  $\tau(p, p', T')$  is approximated by  $\tau(p_{eff}, T_{eff}, w; T_0)$ .

### d. Overlapping with water vapor

In the 540–800 cm<sup>-1</sup> spectral region, there is substantial overlap of the CO<sub>2</sub> absorption with the H<sub>2</sub>O absorption in the troposphere. The total mean transmittance due to both absorbers can be written as

$$\tau_{12} = \int \tau_1(\nu)\tau_2(\nu)d\nu / \int d\nu, \quad (40)$$

where the subscripts 1 and 2 denote the two absorbers. If the transmittance  $\tau(\nu)$  is expressed as the sum of the mean over the spectral band  $\tau$ , and the deviation from the mean  $\tau'(\nu)$ , then (40) reduces to

$$\tau_{12} = \tau_1\tau_2 + \int \tau'_1(\nu)\tau'_2(\nu)d\nu / \int d\nu. \quad (41)$$

A common procedure is to assume that the transmission due to the two absorbers are uncorrelated, i.e., the second term on the right-hand side of (41) is negligible. This term is significant only when the overall shape of the absorption curves due to both of the gases are correlated (or anticorrelated). In the 540–800 cm<sup>-1</sup> spectral band, the water-vapor absorption decreases with increasing wavenumber, but the CO<sub>2</sub> absorption is rather symmetric within the band. Thus, the multiplication approximation  $\tau_{12} \approx \tau_1\tau_2$  can be applied to this band. This result is confirmed by the use of line-by-line calculations.

Water-vapor absorption in the 15- $\mu$ m band is important only in the lower troposphere. Chou (1984) has found that the one-parameter scaling can be used to accurately compute the mean transmittance in the 540–800 cm<sup>-1</sup> region due to water-vapor line absorption. The water-vapor amount is scaled according to (13) with the parameters  $p_r, T_r, m, r_1$ , and  $r_2$  given in Table 2, and the mean diffuse transmittances computed at 550 mb and 256 K are fit by (34) with the coefficients  $a, b$ , and  $n$  given in Table 3. The units of the scaled water-vapor amounts  $w_{eff}$  are grams per square centimeter. For the water-vapor continuum absorption, the mean diffuse transmittance is computed from

$$\tau(u_{eff}) = \exp(-cu_{eff}^k), \quad (42)$$

where  $c$  and  $k$  are regression coefficients, and  $u_{eff}$  is the scaled water-vapor amount (in  $\text{g cm}^{-2}$ ) for continuum absorption given by (15). Values of  $c$  and  $k$  are given in Table 3. Relative errors in the absorptance for both line and continuum absorption are  $<5\%$ . The total transmittance  $\tau_{total}$  due to both  $\text{CO}_2$  and  $\text{H}_2\text{O}$  is then computed from

$$\tau_{total} = \tau(\text{CO}_2)\tau(w_{eff})\tau u_{eff}, \quad (43)$$

where  $\tau(\text{CO}_2)$  is the diffuse transmittance due to  $\text{CO}_2$ .

#### e. Fluxes and cooling rate

Fluxes and cooling rate for a midlatitude summer atmosphere and a  $\text{CO}_2$  concentration of 300 ppmv are computed using the line-by-by method and the three different parameterizations for the  $\text{CO}_2$  transmission functions. For the parameterizations, the total absorption due to the overlapping of  $\text{CO}_2$  and  $\text{H}_2\text{O}$  is computed using the multiplication approximation. Figure 4 shows the cooling rate profiles from line-by-line calculations (solid curve) and the parameterization using the two-parameter scaling [with table look-up and (33)] (dashed curve). The cooling rate errors for the parameterizations are shown in Fig. 5. The param-

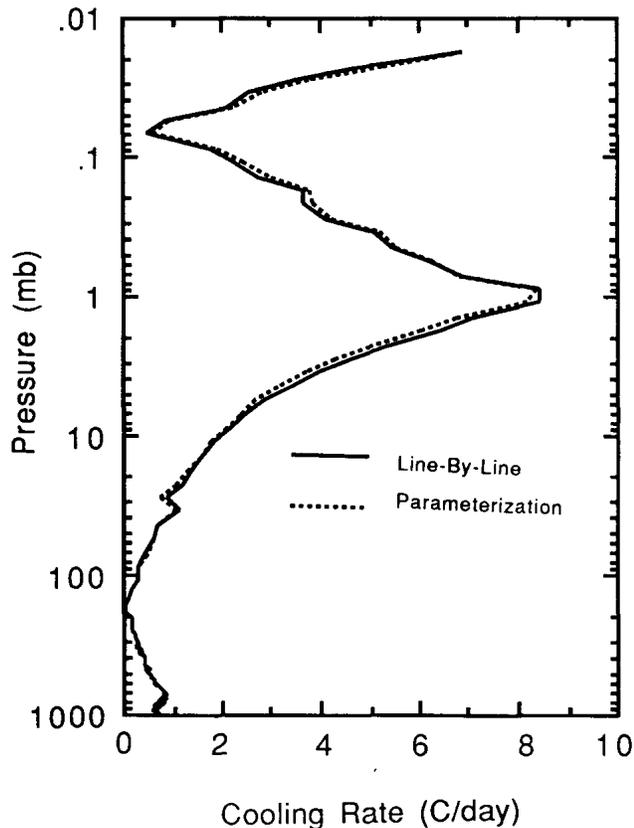


FIG. 4. Cooling rate profiles in the  $15\text{-}\mu\text{m}$  band ( $540\text{--}800\text{ cm}^{-1}$ ) for a midlatitude summer atmosphere. The parameterization uses the two-parameter scaling and the table lookup for the  $\text{CO}_2$  transmission function. Overlapping of  $\text{CO}_2$  and water-vapor line and continuum absorption is taken into account in the calculations.

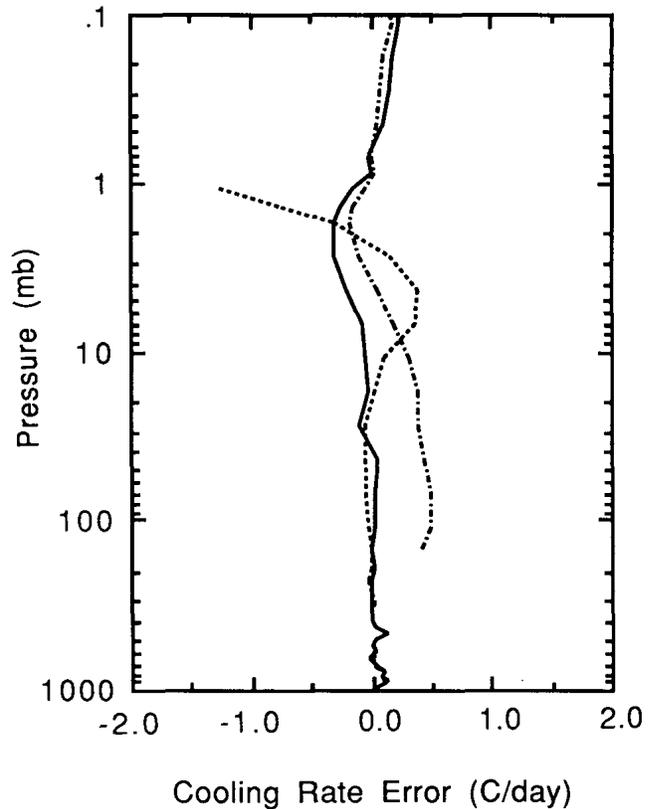


FIG. 5. Cooling rate errors in the  $15\text{-}\mu\text{m}$  band. The solid curve is for the parameterization using the two-parameter scaling and the table lookup for the  $\text{CO}_2$  transmission function, the dashed curve is for the tropospheric parameterization, and the dot-dashed curve is for the middle-atmospheric parameterization.

terization with the two-parameter scaling and table look-up is shown by the solid curve. Throughout the atmosphere from 0.1 mb to the surface the error is  $<0.3^\circ\text{C day}^{-1}$ . The parameterization with the one-parameter scaling and (34) is shown by the dashed curve. The cooling rate is accurate in the troposphere and the lower stratosphere. Above the 2-mb level, the error is unacceptably large. The parameterization using (36)–(39) for the middle atmosphere is shown by the dash-dotted curve. The cooling rate is accurate above the 20-mb level. Fluxes at the top of the atmosphere and the surface computed using the line-by-line method and the parameterizations are shown in Table 1. Compared to line-by-line calculations, the error in fluxes for the approaches using table look-up and the one-parameter scaling is  $\leq 2\%$ . The parameterization for the middle atmosphere is for cooling rate only; the error in fluxes is large and is not shown in the table.

In numerical modeling of the atmosphere, the computing time can be greatly reduced by vectorizing computer codes in certain large-scale computers. To be efficient in vectorizing the codes, analytical functions are preferred to the table look-up for the transmission functions. The two parameterizations for, respectively, the troposphere and the middle atmosphere involve

only analytical functions that can be easily vectorized. Two sets of cooling rates may be separately computed with the two parameterizations and the results merged. As can be seen in Fig. 5, the cooling rate in the region 2–20 mb computed by both methods are accurate. The boundary for merging the two cooling profiles can be chosen at the 10-mb level. Because the accuracy is comparable to the parameterization based on table lookup, the approach of merging two sets of computations may be preferred for certain applications.

## 6. The 9.6- $\mu\text{m}$ O<sub>3</sub> band

Ozone has a significant effect on the stratospheric cooling and, to a lesser extent, the surface flux. Broad-band absorptance has been parameterized by a number of investigators. Recent examples are the parameterizations by Kratz and Cess (1988) and Wang and Shi (1988), which yield accurate total band absorptance. Both studies considered only the pressure broadening of molecular lines. To account for the Doppler effect, Fels (1979) suggested an empirical adjustment to the pressure-broadened line half-width. Following this suggestion, Rosenfield et al. (1987) modified the effective line half-width in computing stratospheric diabatic heating. The accuracy of this approach, however, is limited by applying a random band model to the total O<sub>3</sub> band.

In the O<sub>3</sub> absorption band from 980 to 1100 cm<sup>-1</sup>, variation of the Planck function is small, and the Planck-weighted diffuse transmittance can be approximated by (32). Chou and Kouvaris (1990) showed that the use of the two-parameter scaling and table lookup, as for the 15- $\mu\text{m}$  CO<sub>2</sub> band, could accurately compute the cooling in the 9.6- $\mu\text{m}$  band. The diffuse transmittance was computed from (33) and the three two-dimensional tables of  $\tau(p_{\text{eff}}, T_r, w)$ ,  $\alpha(p_{\text{eff}}, w)$  and  $\beta(p_{\text{eff}}, w)$ .

In this spectral region, water-vapor continuum absorption is important, especially in a humid atmosphere. The diffuse transmission function for continuum absorption is fit by (42) with the coefficients  $c$  and  $k$  given in Table 3. The error in the curve-fitting is <0.002. Water-vapor line absorption is weak and is neglected in the study of Chou (1984). Line-by-line calculations show that the line absorption enhances the downward flux at the surface by 1.3 W m<sup>-2</sup> for the midlatitude summer atmosphere. It is, therefore, desirable to include this effect for accurate surface radiation calculations. As in the 15- $\mu\text{m}$  band, it is found that the mean diffuse transmittance  $\tau(w_{\text{eff}})$  in the 980–1100 cm<sup>-1</sup> region for water-vapor line absorption can be fit well by (34) with the scaled water-vapor amount given by (11). The fractional transmittance error in the curve fitting is <0.001. Values of  $p_r$ ,  $T_r$ ,  $m$ ,  $r_1$ , and  $r_2$  for scaling the water-vapor amount are given in Table 2, and the coefficients  $a$ ,  $b$ , and  $n$  for transmittance calculations are given in Table 3. The units of the scaled

water-vapor amount are grams per square centimeter. The multiplication approximation is used to compute the total transmittance  $\tau_{\text{total}}$  due to both O<sub>3</sub> and water vapor,

$$\tau_{\text{total}} = \tau(\text{O}_3)\tau(w_{\text{eff}})\tau(u_{\text{eff}}), \quad (44)$$

where  $\tau(\text{O}_3)$  is the diffuse transmittance due to ozone.

Figure 6 shows the cooling rate profiles from line-by-line calculations and the parameterization for the midlatitude summer atmosphere. The difference of the two profiles is small (<0.1°C day<sup>-1</sup>) throughout the atmosphere. Fluxes at the top of the atmosphere and the surface are shown in Table 1 for both the midlatitude summer atmosphere and the subarctic winter atmosphere. Compared to line-by-line calculations, the flux error of the parameterization is <0.6 W m<sup>-2</sup>.

## 7. Nitrous oxide and methane bands

Nitrous oxide (N<sub>2</sub>O) and methane (CH<sub>4</sub>) have a number of absorption bands in the thermal infrared. Only those located at 7.6- $\mu\text{m}$  and 17- $\mu\text{m}$  regions have a significant effect on climate. The absorption is due to N<sub>2</sub>O, CO<sub>2</sub>, and H<sub>2</sub>O in the 17- $\mu\text{m}$  region and due to N<sub>2</sub>O, CH<sub>4</sub>, and H<sub>2</sub>O in the 7.6- $\mu\text{m}$  region. In the 17- $\mu\text{m}$  band, the spectral range is narrow and, hence,

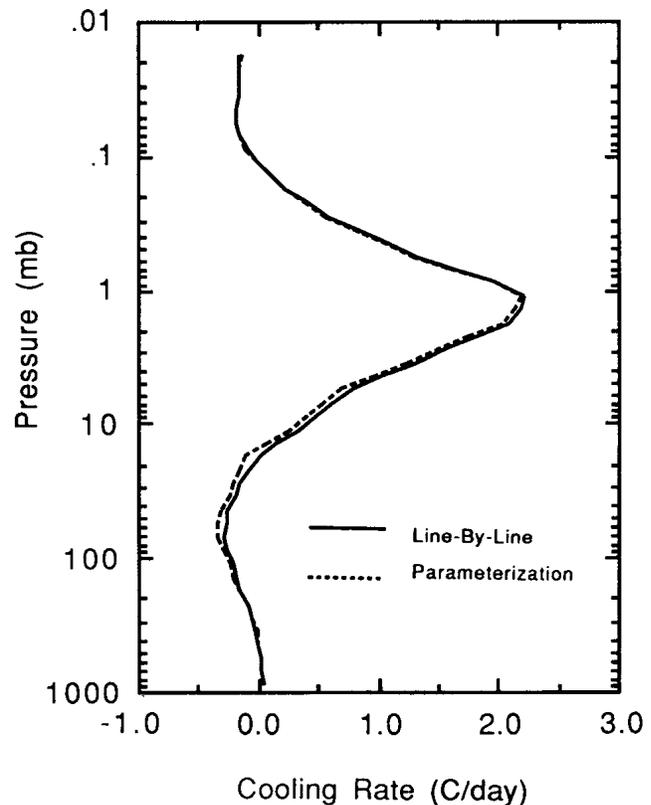


FIG. 6. Cooling rate profiles in the 9.6- $\mu\text{m}$  band (980–1100 cm<sup>-1</sup>) for a midlatitude summer atmosphere. Overlapping of O<sub>3</sub> and water-vapor line and continuum absorption is taken into account in the calculations.

it is relatively easy to parameterize the absorption. On the other hand, parameterization for the absorption in the 7.6- $\mu\text{m}$  region is complicated by the following factors: 1) absorption due to  $\text{N}_2\text{O}$ ,  $\text{CH}_4$ , and  $\text{H}_2\text{O}$  overlap each other. The multiplication approximation appears to be the only practical means for computing the total transmission function. 2) The  $\text{CH}_4$  absorption band covers the region 1215–1380  $\text{cm}^{-1}$ , while the  $\text{N}_2\text{O}$  band is staggered from 1135 to 1340  $\text{cm}^{-1}$ . On the other hand, the absorption due to water vapor has a systematic trend in the region 1135–1380  $\text{cm}^{-1}$ . As a result, the absorption due to the three absorbers are not really randomly correlated. The second term on the rhs of (41) may not be small, and the multiplication approximation could introduce large errors. 3) Within the wide spectral region from 1135 to 1380  $\text{cm}^{-1}$ , the Planck function varies strongly with wavenumber and temperature. The approximation given by equation (32) might introduce large errors.

Donner and Ramanathan (1980) and Cess et al. (1986) developed efficient broadband models for the spectrally integrated absorptance in the  $\text{N}_2\text{O}$  and  $\text{CH}_4$  bands. The spectral regions involved were wide, but the spectral variation of the Planck function was not considered. Because of the previously mentioned difficulties, it is not clear how well these models can be applied to the broad spectral region from 1135 to 1380  $\text{cm}^{-1}$ , where the Planck function and the absorption due to water vapor have a significant spectral trend. To cope with these difficulties, the broad 7.6- $\mu\text{m}$  band is divided into two regions (1135–1215 and 1215–1380  $\text{cm}^{-1}$ ) in this study. With the spectral region narrowed, the Planck-weighted diffuse transmittance  $\tau(p, p', T')$  is approximated by  $\tau(p, p', T_0)$ .

To simplify the parameterization, the one-parameter scaling is used. The scaling parameters  $p_r$ ,  $T_r$ ,  $m$ ,  $r_1$ , and  $r_2$  are given in Table 2. The spectral region covered is 560–615  $\text{cm}^{-1}$  for  $\text{N}_2\text{O}$ ,  $\text{CO}_2$ , and  $\text{H}_2\text{O}$  absorption; 1135–1215  $\text{cm}^{-1}$  for  $\text{N}_2\text{O}$  and  $\text{H}_2\text{O}$  absorption, and 1215–1380  $\text{cm}^{-1}$  for  $\text{N}_2\text{O}$ ,  $\text{CH}_4$  and  $\text{H}_2\text{O}$  absorption. The absorption in the 7.6- $\mu\text{m}$  region is important only in the troposphere, and the reference pressure  $p_r$  is chosen to be the value in the middle troposphere (500 mb). On the other hand, the absorption due to water vapor and  $\text{CO}_2$  in the 17- $\mu\text{m}$  region is strong. Nitrous oxide has a very little effect in the lower troposphere. The reference pressure is, therefore, chosen to be the value in the upper troposphere (300 mb).

Table 4 shows the reduction of fluxes at the tropopause and the surface due to the absorption by, respectively,  $\text{H}_2\text{O}$ ,  $\text{N}_2\text{O}$ , and  $\text{CH}_4$  in the 7.6- $\mu\text{m}$  region. The scaled line-by-line calculation is the same as the "exact" line-by-line calculation, except the absorption coefficient  $k_\nu(p, T)$  is extrapolated from  $k_\nu(p_r, T_r)$  according to (9). The flux error of the one-parameter scaling is negligible at the tropopause and is <5% at the surface.

TABLE 4. Reduction of fluxes at the surface and the tropopause in the 1215–1380  $\text{cm}^{-1}$  region due to the absorption by, respectively, water vapor,  $\text{N}_2\text{O}$  and  $\text{CH}_4$ . The midlatitude summer atmosphere is used. The gas concentration is set to 0.28 ppmv for  $\text{N}_2\text{O}$  and 1.75 ppmv for  $\text{CH}_4$ .

	Tropopause			Surface ( $F\downarrow$ )
	$F\uparrow$	$F\downarrow$	$F_{\text{net}}\uparrow$	
Water vapor				
Exact line-by-line	7.56	-0.02	7.58	-5.66
Scaled line-by-line	7.50	-0.01	7.51	-5.86
Difference	-0.06	0.01	-0.07	-0.20
$\text{N}_2\text{O}$				
Exact line-by-line	2.50	-0.12	2.62	-3.72
Scaled line-by-line	2.48	-0.11	2.59	-3.70
Difference	-0.02	0.01	-0.03	0.02
$\text{CH}_4$				
Exact line-by-line	4.04	-0.18	4.22	-5.85
Scaled line-by-line	3.97	-0.20	4.17	-5.57
Difference	-0.07	-0.02	-0.05	0.28

Note: Overlapping of absorption among gases is not considered in the calculations.

Using the one-parameter scaling, the Planck-weighted diffuse transmission function  $\tau(p, p', T')$  is approximated by  $\tau(w_{\text{eff}}, T_0)$ . Values of  $\tau(w_{\text{eff}}, T_0)$  were computed at  $T_0 = 250$  K for a number of  $w_{\text{eff}}$  using the line-by-line method and then fit by (34). The coefficients  $a$ ,  $b$ , and  $n$  in (34) for computing the diffuse transmittances are given in Table 3. The units of  $w_{\text{eff}}$  are grams per square centimeter for  $\text{H}_2\text{O}$  and  $(\text{cm-atm})_{\text{STP}}$  for  $\text{CO}_2$ ,  $\text{N}_2\text{O}$  and  $\text{CH}_4$ . The relative error in the fractional absorptance using (34) is <5% for relevant amounts of  $\text{CO}_2$ ,  $\text{N}_2\text{O}$ ,  $\text{CH}_4$ , and  $\text{H}_2\text{O}$  found in the atmosphere.

One of the parameterizations for  $\text{N}_2\text{O}$  transmittance is for the 1215–1340  $\text{cm}^{-1}$  region. To be consistent with  $\text{H}_2\text{O}$  and  $\text{CH}_4$ , the mean diffuse transmittance is adjusted for the 1215–1380  $\text{cm}^{-1}$  region by

$$\tau_w = 1 - 0.8072(1 - \tau_n) \quad (45)$$

where the subscript  $w$  denotes the wide region 1215–1380  $\text{cm}^{-1}$  and  $n$  denotes the narrow region 1215–1340  $\text{cm}^{-1}$ . The constant 0.8072 is the ratio of the Planck function at 250 K integrated over the region 1215–1340  $\text{cm}^{-1}$  to that integrated over 1215–1380  $\text{cm}^{-1}$ .

Incorporation of  $\text{N}_2\text{O}$  and  $\text{CH}_4$  absorption in flux calculations usually requires regrouping of spectral regions and, hence, new parameterizations for broadband transmission functions. This requirement can be avoided by computing only the flux changes due to  $\text{N}_2\text{O}$  and  $\text{CH}_4$  absorption. Changes in the upward flux  $\Delta F\uparrow$  and the downward flux  $\Delta F\downarrow$  can be expressed as

$$\Delta F \uparrow(p) = B(T_s) \Delta \tau(p, p_s; T_0) + \int B(T') \partial[\Delta \tau(p, p'; T_0)] / \partial p' dp', \quad (46)$$

$$\Delta F \downarrow(p) = \int B(T') \partial[\Delta \tau(p, p'; T_0)] / \partial p' dp', \quad (47)$$

where

$$\Delta \tau = \tau_1(\tau_2 \tau_3 - 1), \quad (48)$$

and the subscript 1 denotes H<sub>2</sub>O (and CO<sub>2</sub> in the 17- $\mu$ m band), and the subscripts 2 and 3 denote, respectively, N<sub>2</sub>O and CH<sub>4</sub>. Table 5 shows the change in fluxes at the top of the atmosphere, the tropopause, and the surface due to N<sub>2</sub>O and CH<sub>4</sub> absorption. For the parameterization, the change is computed from (46)–(48). Compared to line-by-line calculations, the error of the parameterization is negligible at the tropopause and <0.2 W m<sup>-2</sup> at the surface.

## 8. Summary

Parameterization for IR transmission functions involves two major steps: 1) scaling of a nonhomogeneous path and 2) reduction of the size of the precomputed transmission tables (or introducing band models). Because atmospheric cooling is predominantly contributed from the energy exchange with nearby layers, the two-parameter scaling can be generally applied to a wide range of atmospheric conditions. For absorption bands with a relatively narrow spectral range, the transmission function can then be derived either from small precomputed tables or from analytical functions fitting to line-by-line calculations. For absorption bands with a wide spectral range, the number of independent variables in the transmission function

TABLE 5. Changes in upward fluxes at the top of the atmosphere  $\Delta F \uparrow$  (top), net upward fluxes at the tropopause  $\Delta F_{\text{net}}$  (trop), and the downward fluxes at the surface  $\Delta F \downarrow$  (sfc) for a clear midlatitude summer atmosphere. The absorber concentrations are set to 300, 0.28, and 1.75 ppmv for CO<sub>2</sub>, N<sub>2</sub>O, and CH<sub>4</sub>, respectively. The units of fluxes are watts per square meter.

	$\Delta F \uparrow$ (top)	$\Delta F_{\text{net}}$ (trop)	$\Delta F \downarrow$ (sfc)
560–615 cm <sup>-1</sup> , (H <sub>2</sub> O + CO <sub>2</sub> + N <sub>2</sub> O) – (H <sub>2</sub> O + CO <sub>2</sub> )			
Line-by-line	–0.40	–0.49	0.01
Parameterization	–0.41	–0.45	0.02
1135–1215 cm <sup>-1</sup> , (H <sub>2</sub> O + N <sub>2</sub> O) – H <sub>2</sub> O			
Line-by-line	–0.34	–0.27	0.23
Parameterization	–0.29	–0.25	0.29
1215–1380 cm <sup>-1</sup> , (H <sub>2</sub> O + CH <sub>4</sub> + N <sub>2</sub> O) – H <sub>2</sub> O			
Line-by-line	–3.55	–3.47	1.70
Parameterization	–3.75	–3.48	1.52

increases. In such a case, the parameterization can be simplified by grouping the spectral regions with similar optical properties and separating the middle atmosphere from the lower atmosphere. With the range of relevant pressures narrowed, the one-parameter scaling can then be used to efficiently compute the transmission function.

The transmission function of water vapor is parameterized separately for the middle atmosphere and the lower atmosphere. In the middle atmosphere, the pressure and temperature dependence of absorption is relatively weak. The two-parameter scaling is used to compute the effective pressure and temperature of a path, and the diffuse transmission function is extrapolated from that at a reference pressure and temperature. In the lower stratosphere and the troposphere, the spectrum is grouped into a band-center region and a band-wing region. The one-parameter scaling is applied, and diffuse transmittances are derived from simple functions. For a numerical climate model including both the middle atmosphere and the troposphere, the cooling profile can be obtained by merging the results of the two parameterizations. Sample calculations show that, compared to line-by-line calculations, the error in atmospheric cooling rate is <0.15°C day<sup>-1</sup>.

The CO<sub>2</sub> transmission function is parameterized in three different ways: a general approach applicable to both the middle atmosphere and the lower atmosphere; a middle-atmospheric parameterization; and a tropospheric parameterization. The two-parameter scaling is used for the first two parameterizations, while the one-parameter scaling is used for the third. The diffuse transmission function is derived from precomputed tables in the first approach, and from analytical functions in the last two approaches. The accuracies are found to be comparable for the three parameterizations. The error in atmospheric cooling rate calculations is <0.3°C day<sup>-1</sup>. In the 9.6- $\mu$ m O<sub>3</sub> band, the two-parameter scaling is used, and the transmission function is derived from precomputed tables, as in the CO<sub>2</sub> band. Cooling rate calculations in this band are very accurate. The maximum error is <0.1°C day<sup>-1</sup>.

Other than the 15- $\mu$ m band, there are numerous minor absorption bands which cause climate “greenhouse” warming. Only the absorption due to N<sub>2</sub>O and CH<sub>4</sub> are parameterized in this study. Since the absorption due to these minor gases is weak, the effect is important only in the troposphere. The one-parameter scaling is used, and the diffuse transmission function is computed from a simple function fitting to line-by-line calculations. The parameterization is reduced to such a form that only the change in fluxes due to these minor gases is computed, so that other parts of a radiation routine remains unperturbed.

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